1. Prove that T is one-to-one if and only if $T(x) = 0$ implies $x = 0$.

2. Let $T : V \to W$ be a linear map. Let S be a linearly independent subset of V. Prove that if T is one-to-one, then $T(S)$ is linearly independent. Give a counterexample when T is not one-to-one.

3. Let $T : V \to W$ be a linear map. Let S be a spanning subset of V. Prove that if T is onto, then $T(S)$ is spanning. Give a counterexample when T is not onto.

4. Let $T : V \to W$ and $B = \{v_1, \ldots, v_n\}$ be a basis for V and $\{w_1, \ldots, w_n\} \subseteq W$. Prove that there is exactly one linear transformation such that $T(v_i) = w_i$ for all $i = 1, \ldots, n$. What could go wrong if B is not spanning? What could go wrong if B is not linearly independent.

5. Let $T : V \to W$ and $U : V \to W$. Let B be a basis for V, prove that if $T(b) = U(b)$ for all $b \in B$, $T = U$.

6. Identity the polynomial $ax^2 + bx + c$ with the vector (a, b, c). What is the matrix corresponding to $\frac{d}{dx} : P_2(\mathbf{R}) \to P_2(\mathbf{R})$. What is the kernel? What is the range?

7. What is the kernel of differentation from $C^\infty(\mathbf{R}) \to C^\infty(\mathbf{R})$?