Math 340 Summer '18
Midterm
2018-07-20

Name: ______________________________
Student ID Number: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

- There are 5 problems on this exam. Be sure you have all 5 problems on your exam.
- All vector spaces are defined over a field F which you can take to be $\mathbf{R}$ or $\mathbf{C}$.
- You are allowed a single sheet of 2-sided handwritten self-written notes.
- Use the backsides if you need extra space. Make a note of this if you do.
- Do not cheat. This exam should represent your own work. If you are caught cheating, I will report you to the Community Standards and Student Conduct office.

1. (10 points) Determine whether the following is True or False. You do not need to justify your answer. Write T for True and F for False.

   (a) ____ If S is a spanning subset of V then any subset of S also spans V.

   (b) ____ A linear transformation $T : V \to W$ is one-to-one if and only if the nullity of T is 0.

   (c) ____ If $T : V \to W$ is a linear transformation between infinite dimensional spaces, then $N(T)$ must also be infinite dimensional.

   (d) ____ A linear transformation $T : V \to W$ is one-to-one if and only if $T(0) = 0$.

   (e) ____ Let $T : P_2(\mathbf{R}) \to \mathbf{R}^2$ be a linear map with the property $T(1) = (1, 2)$, $T(x) = (-1, 1)$, and $T(x^2) = (0, 1)$. Then $T(2x^2 + 1) = (1, 4)$.

2. (10 points) Let $T : P_2(\mathbf{R}) \to \mathbf{R}^2$ be a linear transformation (you may assume this) given by

$$T(p) = (p(1), p'(1)).$$

So the first component of $T(p)$ is p evaluated at 1 and the second component is the derivative of p evaluated at 1. Let $\alpha = \{1, x, x^2\}$ be an ordered basis for $P_2(\mathbf{R})$ and $\beta = \{(1,0), (0,1/2)\}$ be an ordered basis for $\mathbf{R}^2$.

(a) What is $[T]_\alpha^\beta$?

(b) Prove that T is onto without using a pivot argument.

(c) What is the nullity of T?

3. (10 points) Let $T : V \to W$ be a linear transformation between vector spaces V and W. Let W_1 be a subspace of W. Prove that $S = \{v \in V : T(v) \in W_1\}$ is a subspace of V.

4. (10 points) Let $T : V \to W$ be a linear transformation between vector spaces V and W. Prove that T is onto if and only, $T(S)$ spans W for any spanning subset S of V.

5. (10 points) Let X, Y, Z be finite dimensional vector spaces. Let $T : X \to Y$ and $S : Y \to Z$ be linear transformations so that

   - T is one-to-one,
   - S is onto,
   - $R(T) = N(S)$.

   Prove that $\dim Y = \dim X + \dim Z$.